Discrete Bulk Reconstruction

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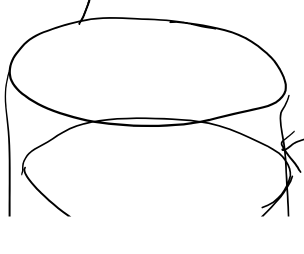
Work-in-progress with Jason Pollack

Rutgers Online Mathematical Physics Seminar

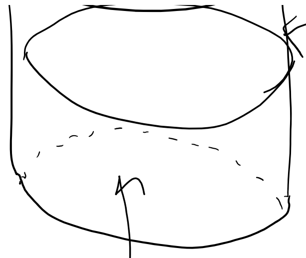
June 6, 2022

WARNING: I'm a theoretical computer scientist who works on quantum computing. What little I know about AdS/CFT, I "picked up on the streets" (or from Jason)!

AdS/CFT (anti-de-Sitter/conformal field theory correspondence): One of the main things to come out of string theory in the past 25 years

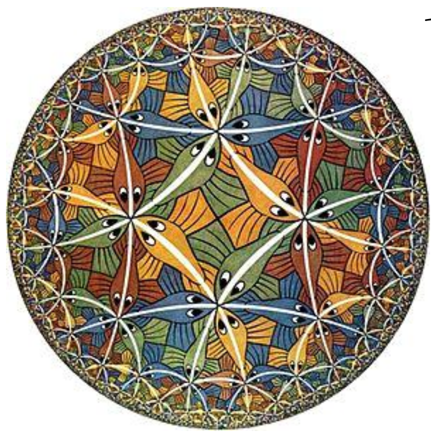


Boundary theory, D-1 spatial dimensions

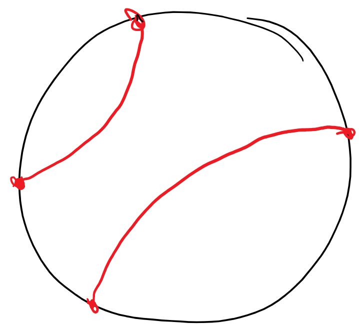


Boundary theory, D-1 spatial dimensions
Fixed geometry
$|\Psi_t\rangle = e^{-iHt}|\Psi_0\rangle$

Bulk theory, D spatial dimensions.
Dynamical geometry, GR, black holes form & evaporate etc.



If vacuum AdS:
Constant negative curvature



GEODESICS

<u>Claim:</u> $\exists$ a "dictionary" mapping states, observables, dynamics in bulk theory to ones in boundary & vice versa.

<u>Caveats:</u>

- Not all boundary states have a "semiclassical bulk dual." E.g., $|v\rangle \to$ Black hole $|w\rangle \to$ No black hole

$$\frac{|v\rangle + |w\rangle}{\sqrt{2}} \; ?$$

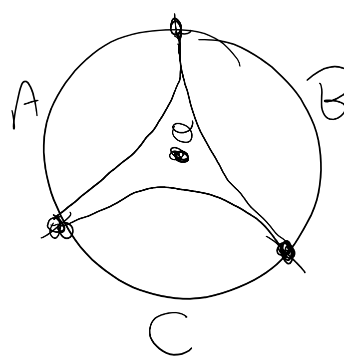
- Not only is "dictionary" not completely known, bulk side lacks an independent definition!
- Obviously, describes a world that's not ours (AdS, unbroken supersymmetry...)

Nevertheless, one of the main toy scenarios where quantum gravity actually seems to work!

"It from Qubit": AdS/CFT & quantum information

Amazing connections over past decade:

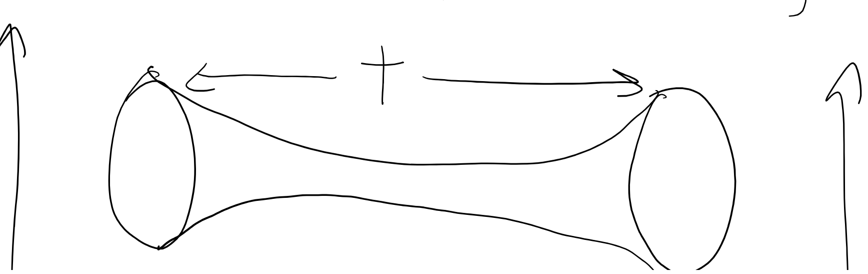
- AdS/CFT as literally an example of a quantum error-correcting code (Almheiri-Dong-Harlow 2014)

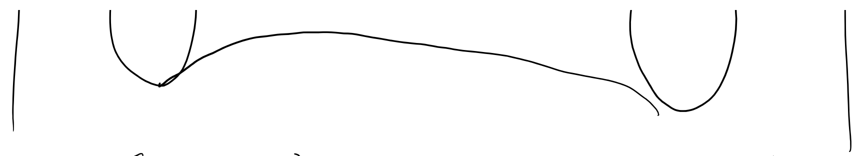


$\otimes$ decodable from AB, BC, or CA but not from A, B, or C alone

- Quantum circuit complexity conjecturally dual to wormhole volume (Susskind 2014)

$|\psi_t\rangle = e^{-iHt}|\psi_0\rangle$





- AdS/CFT and the Quantum Extended Church-Turing Thesis? (Bouland-Fefferman-Vazirani 2019, Susskind 2022)
- AdS/CFT potentially leading to new results in "conventional" quantum information! (May 2019)

<u>Drawback</u>: Still barely comprehensible to QI community. Even when ultimately about qubits, path to get there involves Lorentzian manifolds, path integrals, SUSY, ...

Want to investigate, e.g., under what conditions the bulk/boundary map is computable in polynomial time.

Would be easier if map was inherently discrete!

<u>This Work</u>

- Building on earlier works (e.g. Bao et al. 2015), we study how to proceed directly from a vector of entropies of CFT subregions, to a "graph model" for the bulk

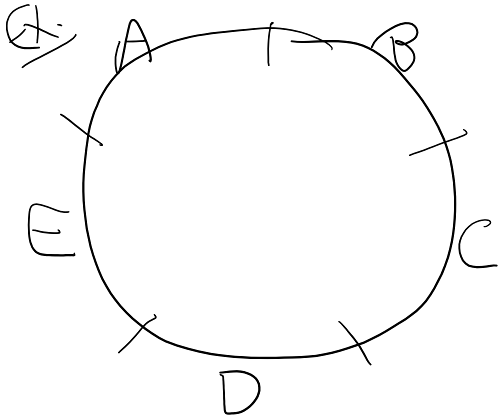
When can this be done in polynomial time? Via a planar graph? With poly(n) vertices? etc

<u>Our Main Result</u>: Given a 1D circular boundary

divided into n discrete regions, and entropies for <u>contiguous</u> unions of those regions, the bulk reconstruction problem is as computationally simple as one could possibly hope.

- $\exists$ planar bulk graph with $O(n^2)$ vertices & edges
- "Universal": only edge weights depend on the entropies, via a linear transformation computable in $O(n^2)$ time.

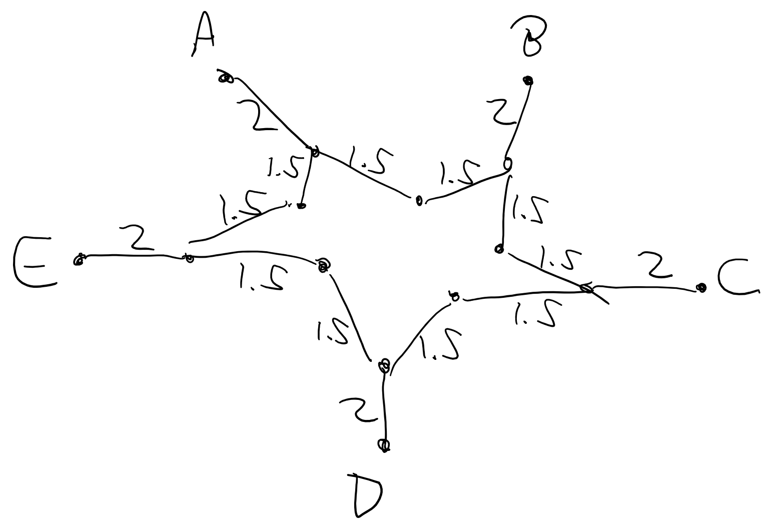
Ex:



$S(A) = S(B) = S(C) = S(D) = S(E) = 2,$

$S(AB) = S(BC) = S(CD) = S(DE) = S(EA) = 3$

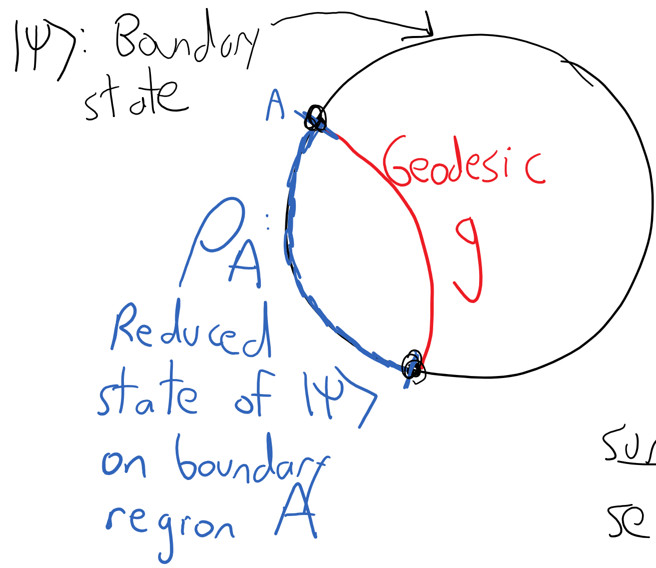
$\Downarrow$



We also prove it's <u>not</u> so simple when there are multiple 1D boundaries, higher-dimensional

boundaries, or non-contiguous boundary regions!
Yields a plethora of well-defined open problems.

Our central tool: the Ryu-Takayanagi (RT) formula (2006)



van Neumann entropy

$$\boxed{\text{length}(g) \propto S(\rho_A)}$$

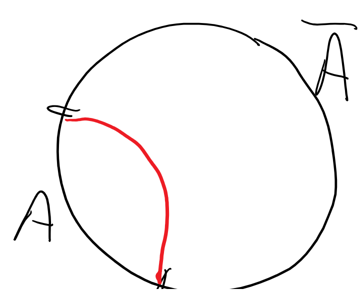
More generally, area of minimal surface in the bulk that separates A from its complement.

{ Notes: There are tiny corrections to RT formula that won't concern us.
Entropy is made finite by energy cutoff or subtracting vacuum contribution.

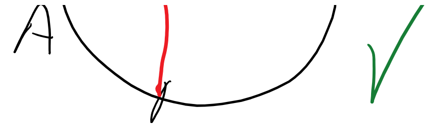
Write $S(A)$, $S(AB)$, etc. instead of $S(\rho_A)$, $S(\rho_{AB})$

Consistency Checks:

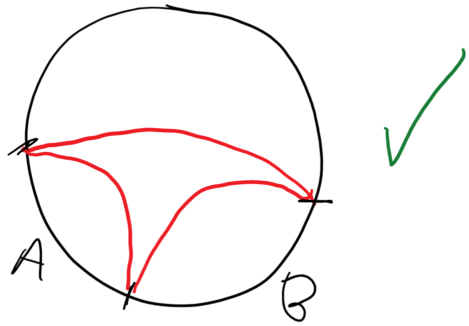
- Complementarity: $S(A) = S(\bar{A})$.



✓

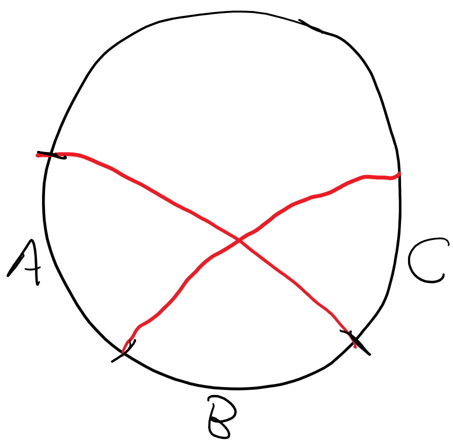


- Subadditivity: $S(A) + S(B) \geq S(AB)$



- Strong Subadditivity (SSA):

$$S(AB) + S(BC) \geq S(B) + S(ABC)$$

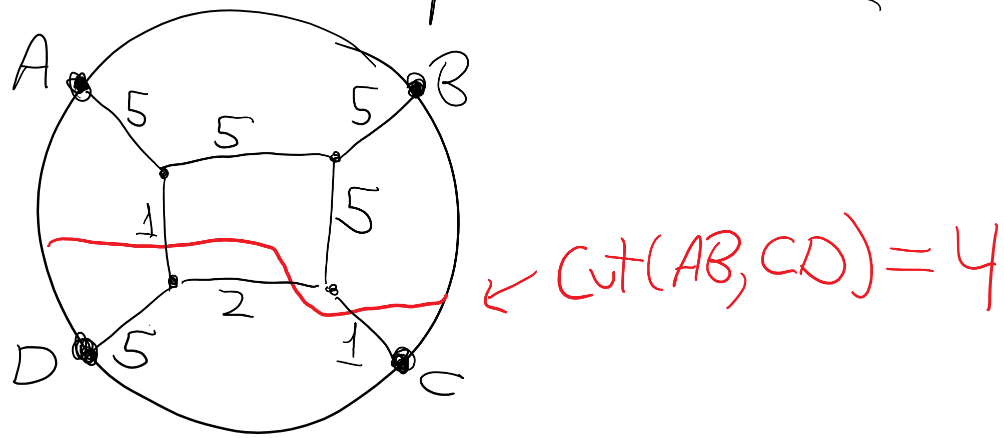


+ Infinitely many additional inequalities characterizing "holographic entropy cone"!
Full list isn't known yet.

Space from Cuts

We model the bulk space as a weighted undirected graph G, with distinguished "boundary" vertices.

Then the RT surfaces correspond to min cuts in G



Finding min cuts:

Famous problem in classical CS, solvable in polynomial time (e.g. via Max-Flow/Min-Cut Theorem).

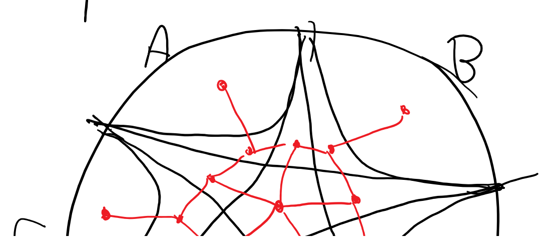
But we now have the inverse problem:

Given prescribed min-cut values (from boundary entropies) is there a graph G consistent with those values? Planar? How many vertices? Findable in $n^{O(1)}$ time?

Observation (Bao et al. 2015): These problems are decidable! Whenever there's a graph, there's one with $\lesssim 2^{2^n}$ vertices, findable in $\sim 2^{2^{2^n}}$ time.

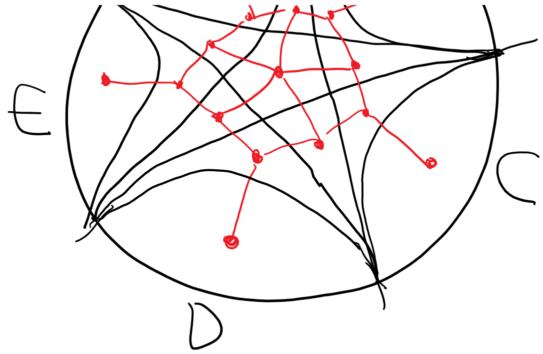
Proof: Construct all 2^{2^n} possible intersections of the 2^n RT regions.

Then w.l.o.g., all



(many intersections

Then w.l.o.g., all vertices in a given intersection can be collapsed to a single vertex. ∎

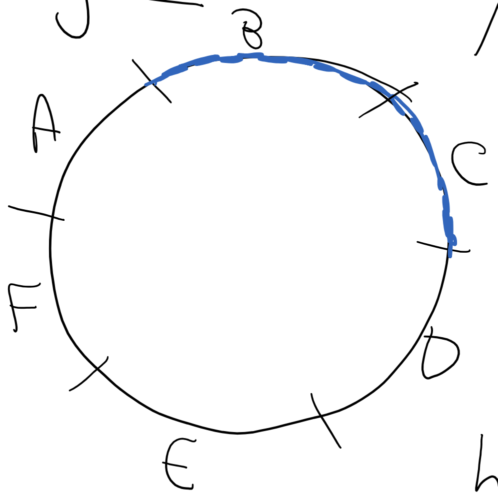


intersections not shown due to planarity)

Naturally we'd like something more efficient!

Our Main Result

Suppose we're given only the $O(n^2)$ entropies for contiguous boundary regions.



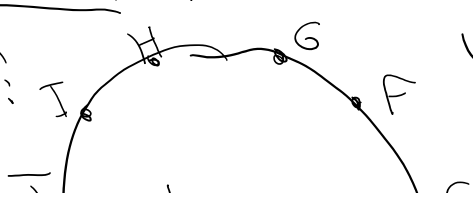
& suppose they satisfy strong subadditivity.

Then a graph model always exists, & it's planar, universal, has $O(n^2)$ vertices & can be found in linear time.

Let a bulkless graph be a graph (not necessarily planar) with only boundary vertices $A, B, C, \ldots$

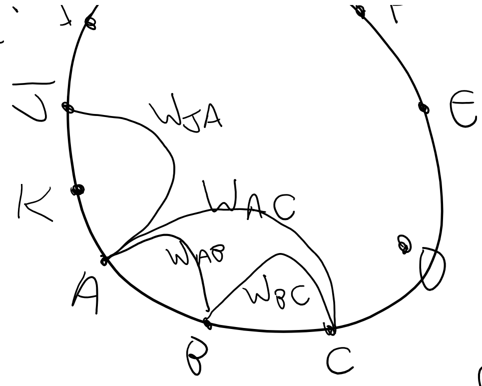
Key Lemma: A bulkless graph model always exists.

Proof:



$$S(AB) = S(A) + S(B) - 2w_{AB}$$

Proof:



$$S(AB) = S(A) + S(B) - 2W_{AB}$$

$$\Rightarrow W_{AB} = \frac{S(A)+S(B)-S(AB)}{2}$$

≥ 0 by subadditivity

& so on for W_{BC}, W_{CD}, etc.

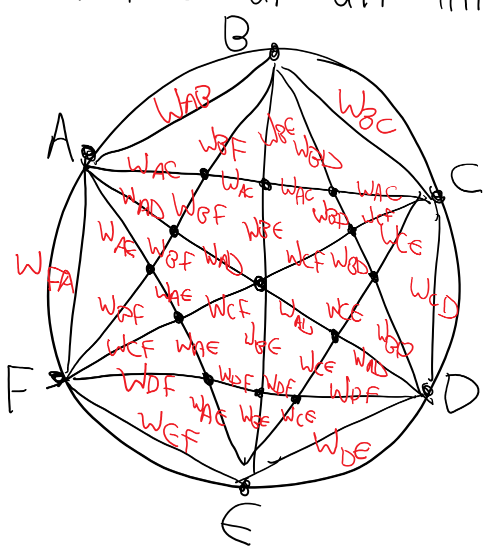
Likewise, $S(ABC) = S(AB) + S(BC) - S(B) - 2W_{AC}$

$$\Rightarrow W_{AC} = \frac{S(AB)+S(BC)-S(B)-S(ABC)}{2}$$

≥ 0 by strong subaddivity, etc.

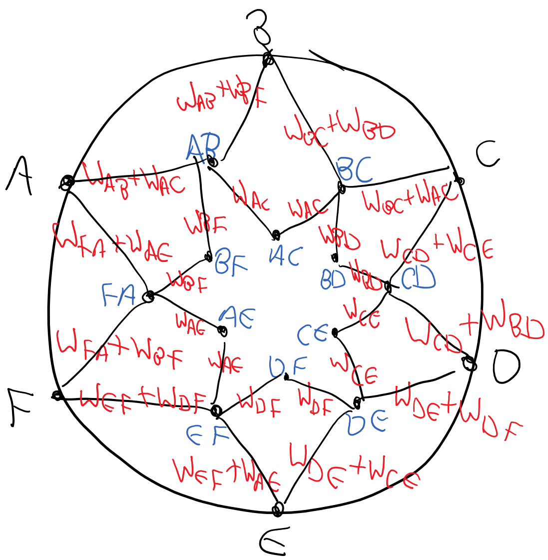
Can solve for all edge weights & they're all nonnegative! ∎

Already yields a planar graph model, by drawing all possible chords, giving them the bulkless weights & putting new vertices at all intersections:



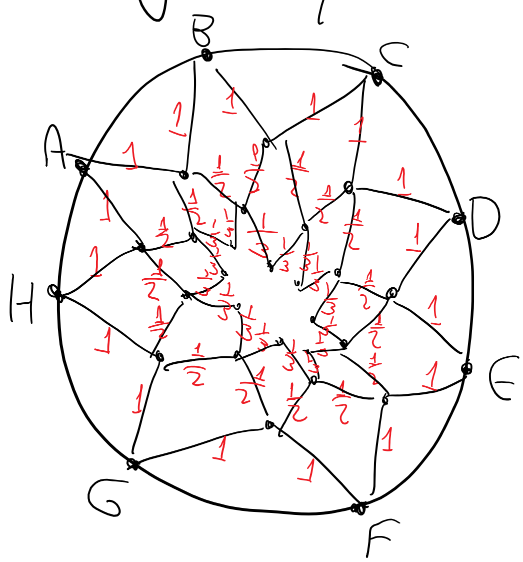
But this has $O(n^4)$ vertices & edges!

More efficient construction: "Diamondwork!"



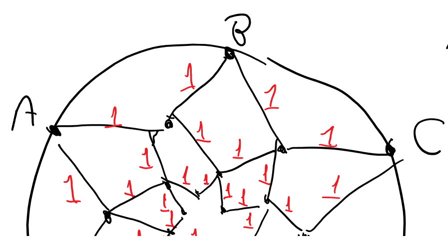
Diamondwork Examples

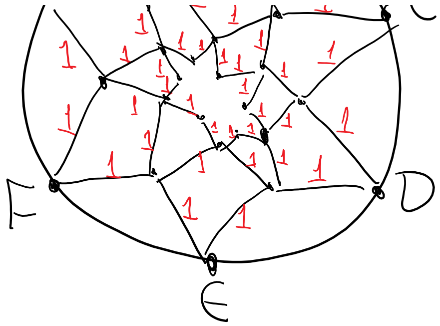
Hyperbolic AdS geometry: harmonic edge weights!



$\Rightarrow$ Geodesic between 2 points L away has length $\sim \ln L$

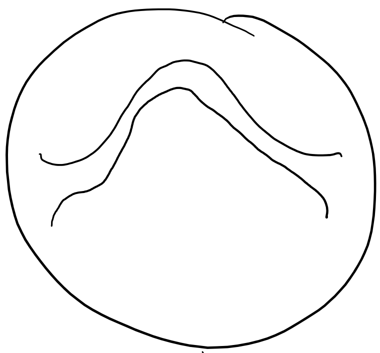
Black hole geometry: edge weights that cause all geodesics to stay near the boundary w.l.o.g.





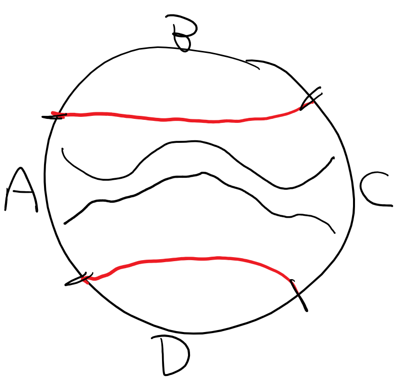
What About Wormholes?

Some bulk geometries in AdS/CFT contain wormholes:



So how is it possible that we always found a planar graph?

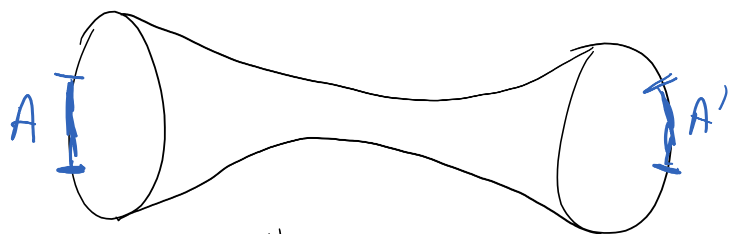
Resolution: Our inputs were entropies of contiguous regions only. To express that there's a wormhole from A to C, one can say, e.g.,



$$S(AC) < S(A) + S(C).$$

Our result shows that there's no way to express this using contiguous entropies only. ✓

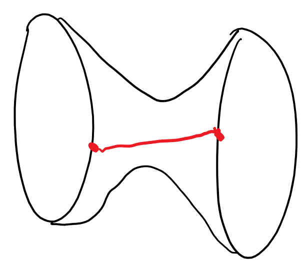
But now suppose there are 2 circular boundaries, and we're given an entropy for every pair of contiguous regions (thus, $O(n^4)$ data points):



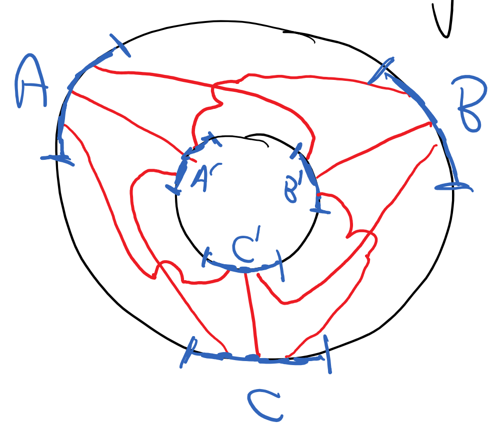
$S(AA') = \ldots$
$S(AB') = \ldots$

As long as this wormhole is long enough that no geodesics "straddle" it, we can generalize our result from the single-boundary case.

But as soon as there are straddling geodesics,



we can embed the non-planar graph $K_{3,3}$:



which implies that there's not always a planar bulk graph. And even when there is, it's not universal. And the set of valid input vectors is not closed under nonnegative linear combination, so not a cone, so there's no list of inequalities like SSA that it suffices to check.

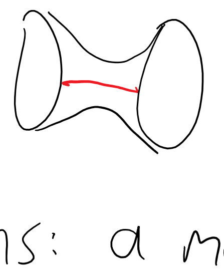
MORE COMPLICATED!

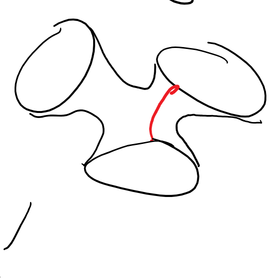
Conclusion & Open Problems

- We've shown that, with a single 1D boundary and contiguous entropies, bulk reconstruction in AdS/CFT is as simple as can be: a linear-time algorithm, working directly on a finite list of real numbers, that builds a weighted undirected graph.

Challenges:

- Generalize to multiple 1D boundaries with straddling geodesics:





- Higher dimensions: a mess!

exp(n) contiguous boundary regions!

What

contiguous boundary regions?
What do we want: a simplicial complex?

- Include dynamics? (RT → HRT formula)
- The general problem of constructing a bulk graph when it exists: can we at least put it in NP or PSPACE?
- What about constructing the CFT state $|\psi\rangle$ from the vector of contiguous entropies?
- Can this picture say anything about why the AdS/CFT dictionary seems to become exponentially complex when extended past a black hole horizon?
- (A personal favorite) For every true inequality of holographic entropies, e.g.

$$S(AB) + S(BC) \geq S(B) + S(ABC),$$

is there a proof based on cutting the left-hand RT surfaces into pieces and then pasting them together?

Thanks for listening!