Adiabatic Quantum Computing

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Quantum Complexity Theory 6.845
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Outline

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The Adiabatic Theorem

Hamiltonian: A hermitian operator with eigenvalues describing the energy eigenstates of the system.
A quantum system with a time-changing Hamiltonian will stay in the same energy level if the rate-of-change is slow enough.

$$T \gg \frac{2\pi\hbar}{\Delta}$$

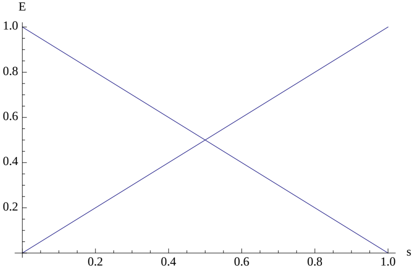
(with level separation Δ).
Consider:

$$H_i = \begin{pmatrix} E_0 & \delta \\ \delta & E_1 \end{pmatrix}, H_T = \begin{pmatrix} E_1 & \delta \\ \delta & E_0 \end{pmatrix}$$

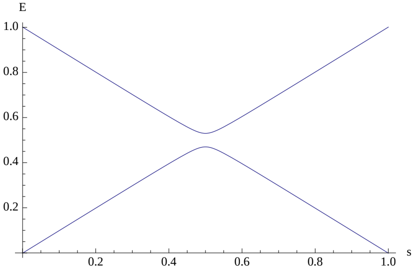
$$H(s) = (1 - s)H_i + sH_T$$

Level Separation

For simplicity, take $E_0 = 0$, $E_1 = 1$.
Plot eigenvalues as a function of $s \in [0, 1]$



$\delta = 0$



$\delta = 0.03$

With s varied over time $T \gg \frac{2\pi\hbar}{\Delta}$, system will remain in the same level.

Outline

The Adiabatic Algorithm

- Encode problem as SAT[1]
- Each clause has a local Hamiltonian H_c encoding the assignment of variables, $H_T = \sum H_c$
- Initialize system into simple ground state of some Hamiltonian H_0.
- Adiabatically evolve Hamiltonian to H_T: system will be in ground state encoding solution

[1] Farhi, et al. 2000.

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- $H(0) = H_0$, $H(T) = H_T$
- $\rightarrow H(t/T = s) = (1 - s)H_0 + sH_T$
- Vary s slowly enough such that system remains in ground state

[1]Farhi, et al. 2000.

Delay Factor

Schrodinger Equation

$$i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

Introduce a Delay Factor $\tau(s)$ indicating how slowly the Hamiltonian varies[2].

$$\frac{d}{ds}|\psi(s)\rangle = -i\tau(s)H(s)|\psi(s)\rangle$$

Adiabatic evolution requires:

$$\tau(s) \gg \frac{\|\frac{d}{ds}H(s)\|_2}{g(s)^2}$$

Evolution time T proportional to separation g_{min}^{-2}.

[2]Van Dam, Mosca, Vazirani, 2008.

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1. The Adiabatic Theorem
2. The Adiabatic Algorithm
3. Computational Complexity
4. Notes on Adiabatic Universality

Adiabatic Complexity

Does not in general indicate 3-SAT soluble in polynomial time: some problems have g_{min} exponentially small

- But can recover Grover Search[3]
- Consider

$$f(x) : \{0,1\}^n \to \mathbb{R} = \begin{cases} 0 & \text{if } x \text{ is the solution} \\ 1 & \text{otherwise} \end{cases}$$

$$\text{Final Hamiltonian } H_x = \sum_{z \in \{0,1\}^n \setminus \{x\}} |z\rangle\langle z|$$

[3] Van Dam, Mosca, Vazirani, 2008.

Grover Search Complexity

Initial Hamiltonian with Hadamard $|+\rangle^n = |\hat{0}\rangle^n$ as ground state.

$$H_0 = \sum_{z \in \{0,1\}^n \setminus \{0^n\}} |\hat{z}\rangle\langle\hat{z}|$$

Level separation of $H(s) = (1-s)H_0 + sH_x$

$$g(s) = \sqrt{\frac{N + 4(N-1)(s^2 - s)}{N}}$$

Looks like $T \propto g_{min}^{-2} = O(N)$. But if we let the delay vary in time:[4]

$$T = \int_{s=0}^{1} \frac{ds}{g(s)^2} = \frac{N \cdot \arctan(\sqrt{N-1})}{\sqrt{N-1}} = O(\sqrt{N})$$

[4]Van Dam, Mosca, Vazirani, 2008.

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Adiabatic Universality

Can show that we can polynomially simulate a general Quantum Circuit adiabatically

- Quantum Circuit: L gates, state after gate ℓ is $|\alpha(\ell)\rangle$
- Adiabatically, could have final Hamiltonian H_L with g.s. $|\alpha(L)\rangle$

Problems: Can't always specify H_L without knowing $|\alpha(L)\rangle$

Adiabatic Universality

Kitaev's history state

$$|\eta\rangle = \frac{1}{\sqrt{L+1}} \sum_{\ell=0}^{L} |\alpha(\ell)\rangle \otimes |1^l 0^{L-\ell}\rangle$$

- Define final Hamiltonian H_f to have $|\eta\rangle$ as ground state.[5]
- Initial Hamiltonian H_0 has g.s. $|\alpha(0)\rangle \otimes |0^L\rangle$
- Can construct H_0 and H_f without knowing $|\alpha(L)\rangle$
- Measure: If clock is $|1^\ell\rangle$ then other register carries result
- Can simulate a QC using 5-local Hamiltonians in $O(L^5)$

Can generalize to 3-local H, or a grid with two-local H and six-state particles.

[5] Aharonov, et al. 2008.

Summary

Conclusions

- Can implement quantum computations by adiabatic evolution
- Slowness of evolution related to complexity of the problem (not known in general)
- Recover $O(\sqrt{N})$ of Grover
- Can implement any quantum circuit adiabatically