Homework 3

Introduction to Quantum Information Science
Due Sunday, February 12 at 11:59 PM

1. Separable and Entangled States. Determine whether each of the following two qubit states are separable or entangled. If the state is separable then provide its factorization into a pair of one qubit states. If the state is entangled then explicitly prove that no factorization into one qubit states exists. Then for each state calculate the local density matrices (often referred to as reduced density matrices) corresponding to each of the qubits individually.

$$|\psi\rangle = \frac{|00\rangle + i\,|01\rangle + i\,|10\rangle - |11\rangle}{2} \qquad |\phi\rangle = \frac{3}{5}\,|01\rangle - \frac{4}{5}\,|10\rangle \qquad |\chi\rangle = \frac{|00\rangle - |01\rangle + |11\rangle}{\sqrt{3}}$$

2. Local Evolution of Entangled States. Suppose Alice and Bob share the two qubit entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Suppose that Alice applies a one qubit unitary transformation U to her qubit, show that this has exactly the same effect as if Bob had applied the unitary transformation $U^\top$ (not the conjugate-transpose of U, just the transpose) to his qubit.

3. Distinguishability of Mixed States Let ρ and σ be two different density matrices. Prove that ρ and σ are distinguishable – that is, there's some measurement basis such that the probabilities of the outcomes is different in the two cases. *Hint: Use the fact that $\sigma - \rho$ is necessarily a nonzero Hermitian matrix.*

4. No-Communication Theorem. Suppose Alice and Bob share the entangled state $\sum_{i=0}^{N-1}\sum_{j=1}^{N-1} \alpha_{ij}\,|i\rangle\,|j\rangle$.

a) What is Bob's local density matrix?

b) Show that Bob's local density matrix is unchanged is Alice measures her subsystem in the standard basis $\{|0\rangle, |1\rangle, \cdots, |N-1\rangle\}$.

c) Show that Bob's local density matrix is also unchanged if Alice applies any $N \times N$ unitary matrix U to her subsystem.